SESSION 1.4

Division with Remainders

MATH FOCUS POINTS

- Using and interpreting division notation
- Solving division problems
- Making sense of remainders in terms of the problem context
- Developing fluency with division facts

TODAY'S PLAN	MATERIALS
10 Class **TEN-MINUTE MATH: REVIEW AND PRACTICE** ***What Time Is It?***	Teacher Presentation
20 Class Pairs **1 ACTIVITY** **Introducing Problems with Remainders**	Teacher Presentation Connecting cubes (14 per pair, optional)
40 **2 MATH WORKSHOP** **Dividing with Remainders** **2A** Problems with Remainders **2B** Practicing Division Facts	**2A** *Student Activity Book,* pp. 475–476 Blank paper (as needed) **2B** C42–C46* Division Cards, Set 1 (from Session 1.2) Scissors (1 per student) Resealable plastic bags or envelopes (as needed)
SESSION FOLLOW-UP: REVIEW AND PRACTICE **Daily Practice and Homework**	*Student Activity Book*, pp. 477–478

* See *Materials to Prepare* in the Investigation 1 Planner.

Common Core State Standards		
	Ten-Minute Math: 3.MD.A.1 **Session:** 3.OA.A.3, 3.OA.C.7	**Daily Practice:** 3.MD.A.2

TEN-MINUTE MATH: REVIEW AND PRACTICE

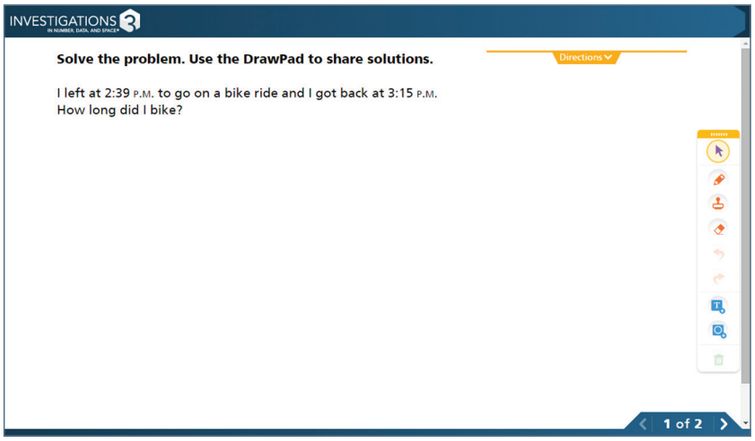
What Time Is It?

10

MATH FOCUS POINTS

- Telling time to the minute on a digital or analog clock
- Determining intervals of time to the minute
- Solving problems involving addition or subtraction of time intervals in minutes

Display the Teacher Presentation or tell students the following story.



I left at 2:39 P.M. to go on a bike ride and I got back at 3:15 P.M. How long did I bike?

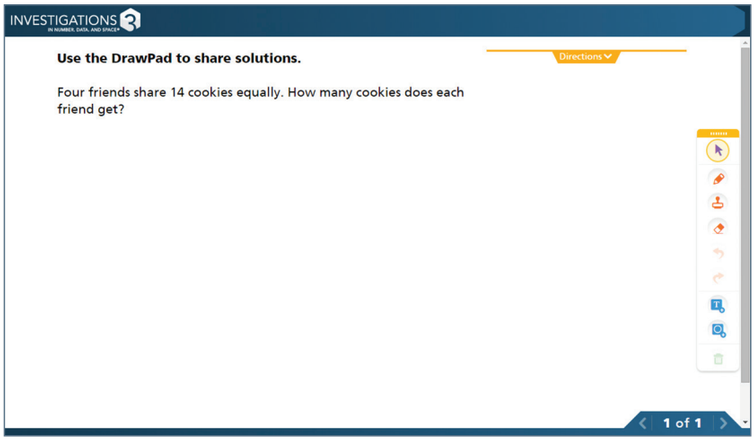
In pairs, students share their solutions. As a class, make sure that students can visualize this situation in which they are determining an interval, not an ending time. Ask students to share their solutions with the class, which may include: adding up from 2:39 to 3:00 and then from 3:00 to 3:15, or subtracting back from 3:15 to 3:00, then from 3:00 to 2:39. Ask similar questions using 1:42 as the starting time and 2:21 as the ending time.

1 ACTIVITY

Introducing Problems with Remainders

20

Display the Teacher Presentation or write the problem. TN



What division problem could we write to represent the situation in this word problem?

If students seem unsure, ask them what parts of the problem they know (the number in all the groups, and the number of groups) and how that could be written using the division sign. Display:

$$14 \div 4$$

Work with a partner to solve and answer this problem. You have to use a drawing or other tool to explain how you got your solution.

Give students a few minutes to solve this problem. While students work, look for solutions in which students share 12 whole cookies among the friends and have 2 leftover cookies, and solutions in which students divide the 2 extra cookies in half so that each friend ends up with $3\frac{1}{2}$ cookies. If both solutions are not represented in your class, introduce the missing one yourself as part of the discussion.

Ask students to share their representations and solutions. PD

STUDENTS MIGHT SAY

"We drew 4 big circles for the friends, and then started sharing the cookies. We used little circles to show that each friend gets 3 cookies. But there are 2 left. We decided to let the teacher have them!"

TEACHING NOTE

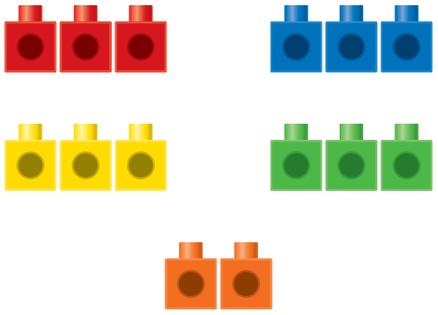
TN Why Remainders in Grade 3?
Although solving division problems with remainders is emphasized in Grade 4, it is important that Grade 3 students are exposed to such problems. Whenever possible, students should encounter different problem types that are accessible to them when they are working with any operation. Otherwise, they generalize incorrectly based on their restricted experience—in this case, that division problems in school always "come out even." Sometimes, such generalizations are unavoidable. For example, most third graders assume that a sum must be equal to or greater than any of the addends that compose it because they are working in the realm of positive numbers. In this case, division problems that result in remainders are well within students' own experience.

PROFESSIONAL DEVELOPMENT

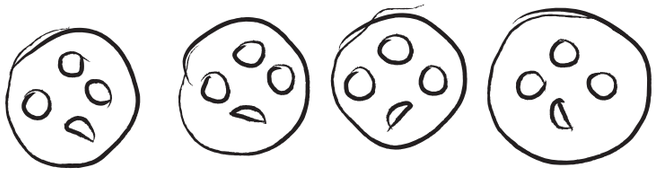
PD TEACHER NOTE 5: What Do You Do with the Remainder?

STUDENTS MIGHT SAY

"We did the same thing, only we used cubes. We got the same answer—each friend gets 3 cookies. There are 2 left, and we weren't sure what to do with them."



"We started the same way as everyone else, each of the friends gets 3 cookies. But then Beatriz remembered that when we were sharing brownies, it was OK to divide up a brownie. So we decided each person could get $\frac{1}{2}$ more cookie—or $3\frac{1}{2}$ cookies!"



Ask if there are any questions about either solution, making certain students agree that "3" and "$3\frac{1}{2}$" are correct answers to the word problem.

So we could get two different answers to this problem. If you don't want to split up the extra cookies, then each friend can only get 3 cookies, and you have 2 leftover cookies. Or you can cut the extra 2 in half and then each friend gets $3\frac{1}{2}$ cookies.

Display (under the word problem):

$$3 \text{ cookies each} \qquad 3\tfrac{1}{2} \text{ cookies each}$$

Let's think about how to notate this problem, starting with the solution in which each friend gets $3\frac{1}{2}$ cookies. Earlier we agreed we were solving 14 divided by 4. How could you write the problem and the answer?

If students seem uncertain, remind them of what information they knew, and what information they were finding (the number in each group). Display:

$$14 \div 4 = 3\tfrac{1}{2}$$

Use the notation and one of the representations to make certain students can explain what each number represents.

What does the 14 mean? (the number in all the groups—how many cookies) **Where do you see it in [Abdul's] representation? What does the 4 mean?** (the number of groups—the people getting cookies) **Where do you see dividing by 4 in the representation? What does the $3\frac{1}{2}$ mean?** (the number in each group—or the number of cookies each person gets)

Let's look at the other answer many of you got—3 cookies for each friend, with 2 left over.

Before asking how to write an answer, use one of the representations to make certain students can explain what each number represents. Ask the same questions as above about the 14, 4, and 3, and where each can be seen in the representation. Then ask about the 2.

What does the 2 mean? Where do you see it in [Elena's] representation?

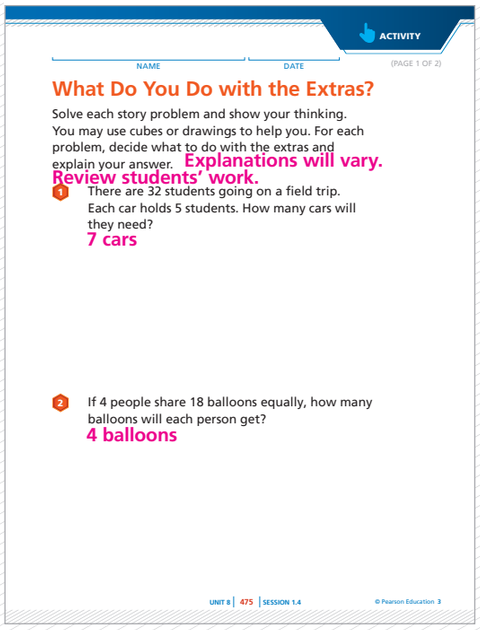
Earlier you agreed we could write $14 \div 4 = 3\frac{1}{2}$. Can I also write $14 \div 4 = 3$?

Display:

$$14 \div 4 = 3$$

What do you think? Talk to a neighbor. One thing that might help you think about this is the relationship between multiplication and division.

STUDENT ACTIVITY BOOK, P. 475



ACTIVITY

NAME DATE (PAGE 1 OF 2)

What Do You Do with the Extras?

Solve each story problem and show your thinking. You may use cubes or drawings to help you. For each problem, decide what to do with the extras and explain your answer. Explanations will vary. Review students' work.

1. There are 32 students going on a field trip. Each car holds 5 students. How many cars will they need?
   7 cars

2. If 4 people share 18 balloons equally, how many balloons will each person get?
   4 balloons

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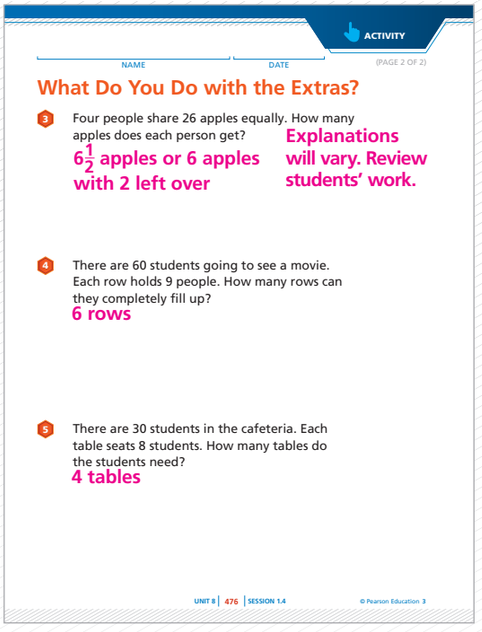
STUDENT ACTIVITY BOOK, P. 476



ACTIVITY

NAME DATE (PAGE 2 OF 2)

What Do You Do with the Extras?

3. Four people share 26 apples equally. How many apples does each person get?
   $6\frac{1}{2}$ apples or 6 apples with 2 left over
   Explanations will vary. Review students' work.

4. There are 60 students going to see a movie. Each row holds 9 people. How many rows can they completely fill up?
   6 rows

5. There are 30 students in the cafeteria. Each table seats 8 students. How many tables do the students need?
   4 tables

UNIT 8 | 476 | SESSION 1.4 © Pearson Education 3

STUDENTS MIGHT SAY

"I think so! We solved the problem and the answer is 3, so 14 ÷ 4 = 3."

"I'm not so sure I agree. If we wrote it as multiplication, it would have to be 'what times 4 = 14?' Then it would be 3 × 4 = 14, but that's not right because 3 × 4 = 12."

"I don't get this, because the problem has two answers, but I don't think you can write two different answers for the same equation."

[Edwin] is right—if we use the equals sign it shows whatever is on each side is equivalent, so we can't write 14 ÷ 4 = 3—we'd have to write 14 ÷ 4 = $3\frac{1}{2}$.

But in some division problems, there are some leftovers. Let's say you didn't want to divide those extra cookies in half and you kept them as leftovers. Then there are other ways you can write the problem and the answer. One way is to answer the word problem and write "3 cookies". One way to answer 14 ÷ 4 is to write 3 with 2 left over, or 3 with 2 extra. TN PD

Here's another question for you. Instead of cookies, what if the 4 friends were sharing 14 balloons? Look at what we said the answers for the cookie problem could be. Would these same answers work for balloons?

Students should recognize that the remaining two balloons cannot be shared equally, so each friend would get 3 balloons and there would be two extra. Three and a half balloons is not a correct answer to the balloon problem.

During Math Workshop you are going to solve some word problems like these, where you have a certain number of equal-sized groups, and some extras. You have to really pay attention to the context to decide how to answer the word problem. Three and a half made sense as an answer to the cookie problem, but not for the balloons. MWI

2 MATH WORKSHOP

Dividing with Remainders

40

In Math Workshop you're going to solve more division problems with leftovers. You're going to practice with your Division Cards, and get a new set of cards that you're going to cut apart. These new cards are the rest of the division facts—today, your only task is to cut them apart.

TEACHING NOTE

TN Using "R" for Remainders One convention for recording division that results in a remainder is "R." For example, for 14 ÷ 4, one way to notate the answer is 3 R2. This notation can be interpreted to mean that there are 3 groups of 4 in 12 plus 2 extra, or 14 ÷ 4 = (4 × 3) + 2. For these sessions in Grade 3, the focus should be on students' making sense of the story context, and answering the question posed by the word problem. Understanding and using notation for remainders in division problems is introduced and is an expectation in Grades 4 and 5. If your students are aware of the notation, have a brief discussion about its use. However, it is incorrect to write 14 ÷ 4 = 3 R2 because 3 R2 does not indicate a specific number (You could not place 3 R2 on a number line.).

PROFESSIONAL DEVELOPMENT

PD **DIALOGUE BOX 1:** What Do You Do with the Extra?

MATH WORDS AND IDEAS

MWI Remainders: What Do You Do With the Extras?

2 A Problems with Remainders

Students solve division story problems on *Student Activity Book* pages 475–476, where each of the answers involves a remainder. Encourage students to use drawings or other math tools for these problems. MPN1 MPN2

ONGOING ASSESSMENT Observing Students at Work

Students work on math problems that result in remainders and consider what to do with the remainder, based on the story context. Tell students that Problems 1 and 4 will be discussed in the next session.

- **How are students solving the problem?** Are they using math tools or drawings to help them make sense of the context?
- **Are students able to determine what to do with the remainder in the context of the problem?** Do they recognize when the answer to the word problem has to be the next whole number (How many cars? How many tables?), or the whole number without "leftovers" (How many balloons?)?

DIFFERENTIATION Supporting the Range of Learners

INTERVENTION **Suggest a Tool** Encourage students to visualize and draw the stories, and identify the quotients and the remainders in their representations.

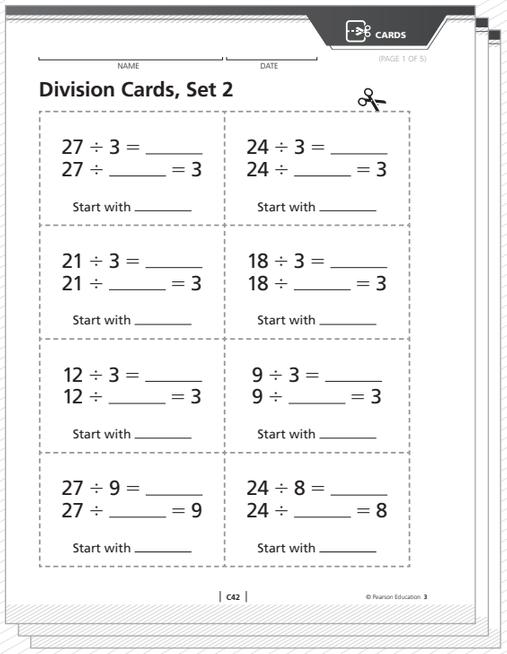
How would drawing a representation help you? [for Problem 1] **I see you have 6 cars, with 5 students in each one. What about the other 2 students? What happens to them? How can they go on the field trip? What do you think we'd do in that situation?**

EXTENSION **Extend Thinking** Students who finish early may work with a partner to share solutions and discuss what they decided to do with the remainder in each situation. Ask these students to see if they can use fractions to divide the remaining two apples in Problem 3. They may also make up word problems that involve remainders for each other to solve.

For a more comprehensive extension activity to be done outside of class, see *What to Do with Leftovers?* at the end of this investigation.

ENGLISH LANGUAGE LEARNERS **Provide Vocabulary Support** Have students use pictures or objects to help them envision each situation. Seeing the story acted out helps students to understand the context and the vocabulary. Ask questions to help them explain what to do with the extras. **What should they do so that the extra people can go? Can you cut a balloon in half? How many rows are completely filled?**

STUDENT ACTIVITY BOOK C42–C46


CARDS

NAME DATE (PAGE 1 OF 5)

Division Cards, Set 2

27 ÷ 3 = _____ 27 ÷ _____ = 3 Start with _____	24 ÷ 3 = _____ 24 ÷ _____ = 3 Start with _____
21 ÷ 3 = _____ 21 ÷ _____ = 3 Start with _____	18 ÷ 3 = _____ 18 ÷ _____ = 3 Start with _____
12 ÷ 3 = _____ 12 ÷ _____ = 3 Start with _____	9 ÷ 3 = _____ 9 ÷ _____ = 3 Start with _____
27 ÷ 9 = _____ 27 ÷ _____ = 9 Start with _____	24 ÷ 8 = _____ 24 ÷ _____ = 8 Start with _____

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MATH PRACTICE NOTES

MPN1 **MP1 Make sense of problems and persevere in solving them.** Students encounter a new kind of problem in this session—division problems with remainders. Help students focus on making sense of these problems by thinking through each part of the problem, picturing what is happening, and making representations as needed to show the relationships between quantities. Not only do students need to solve the problem numerically, but they need to determine what the answer to the problem is in context. For example, when 30 students travel in cars, 4 to a car, for a field trip, the number of cars needed is 8, not 7 R2. In this context, the 2 students would not be left behind.

MPN2 **MP5 Use appropriate tools strategically.** Throughout this year, students have been using a variety of representations to make sense of and solve problems. For these division problems, some students may easily visualize the situation, understand the relationships between quantities, and "just know" the answers. Students need not represent every problem with a picture or model, but should be able to explain to others why their answers make sense, using a representation as needed to justify their solutions.

2 B Practicing Division Facts

Students can practice individually with a set of Division Cards, or work with a partner. At this point, most students should have a small pile of cards they're "working on." As you circulate, check in with students, asking them about cards in each of their piles to make certain they have sorted the cards correctly.

For full details about this activity, see Session 1.2.

Tell students they should cut apart Division Cards, Set 2 (C42–C46), but do nothing else with them in this session.

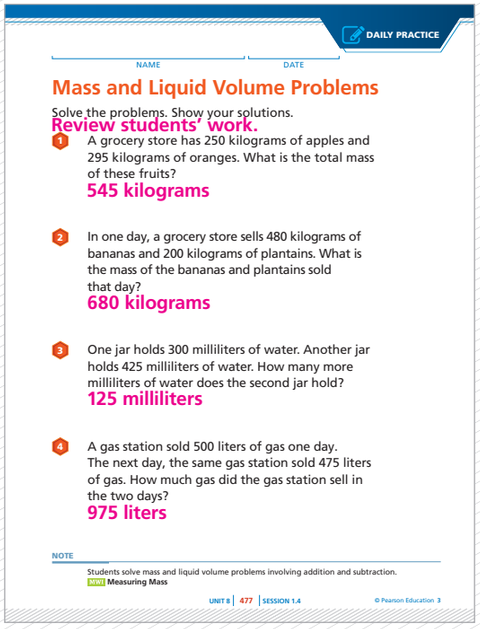
SESSION FOLLOW-UP: REVIEW AND PRACTICE

Daily Practice and Homework

DAILY PRACTICE For ongoing review, have students complete *Student Activity Book* page 477.

HOMEWORK Students solve division problems on *Student Activity Book* page 478.

STUDENT ACTIVITY BOOK, P. 477



DAILY PRACTICE

NAME DATE

Mass and Liquid Volume Problems

Solve the problems. Show your solutions.

Review students' work.

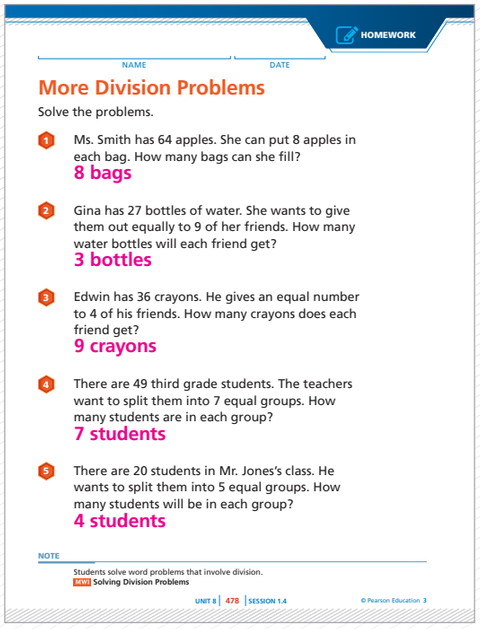
1. A grocery store has 250 kilograms of apples and 295 kilograms of oranges. What is the total mass of these fruits?
   545 kilograms
2. In one day, a grocery store sells 480 kilograms of bananas and 200 kilograms of plantains. What is the mass of the bananas and plantains sold that day?
   680 kilograms
3. One jar holds 300 milliliters of water. Another jar holds 425 milliliters of water. How many more milliliters of water does the second jar hold?
   125 milliliters
4. A gas station sold 500 liters of gas one day. The next day, the same gas station sold 475 liters of gas. How much gas did the gas station sell in the two days?
   975 liters

NOTE

Students solve mass and liquid volume problems involving addition and subtraction.
NEW! **Measuring Mass**

UNIT 8 | 477 | SESSION 1.4 © Pearson Education 3

STUDENT ACTIVITY BOOK, P. 478



HOMEWORK

NAME DATE

More Division Problems

Solve the problems.

1. Ms. Smith has 64 apples. She can put 8 apples in each bag. How many bags can she fill?
   8 bags
2. Gina has 27 bottles of water. She wants to give them out equally to 9 of her friends. How many water bottles will each friend get?
   3 bottles
3. Edwin has 36 crayons. He gives an equal number to 4 of his friends. How many crayons does each friend get?
   9 crayons
4. There are 49 third grade students. The teachers want to split them into 7 equal groups. How many students are in each group?
   7 students
5. There are 20 students in Mr. Jones's class. He wants to split them into 5 equal groups. How many students will be in each group?
   4 students

NOTE

Students solve word problems that involve division.
NEW! **Solving Division Problems**

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SESSION 1.5

Division Facts and Multiplication and Division Practice, Part 1

MATH FOCUS POINTS

- Making sense of remainders in terms of the problem context
- Solving multiplication and division problems
- Developing fluency with division facts

TODAY'S PLAN

TEN-MINUTE MATH: REVIEW AND PRACTICE

10 Class

What Time Is It?

1 DISCUSSION

15 Class

What Do You Do with the Extras?

2 MATH WORKSHOP

45

Multiplication and Division

2A Division Problems

2B Multiplication and Division Story Problems

2C Division Facts 1

SESSION FOLLOW-UP: REVIEW AND PRACTICE

Daily Practice and Homework

MATERIALS

- Teacher Presentation
- *Student Activity Book,* pp. 475–476 (from Session 1.4)
- **2A** *Student Activity Book*, pp. 479–481
- **2B** *Student Activity Book*, pp. 482–483
- **2C** A79*
- A80*
- Division Cards, Sets 1 and 2 (from Sessions 1.2 and 1.4)
- *Student Activity Book,* pp. 484–485

* See *Materials to Prepare* in the Investigation 1 Planner.

Common Core State Standards	**Ten-Minute Math:** 3.MD.A.1 **Session:** 3.OA.A.3, 3.OA.C.7	**Daily Practice:** 3.OA.C.7

TEN-MINUTE MATH: REVIEW AND PRACTICE

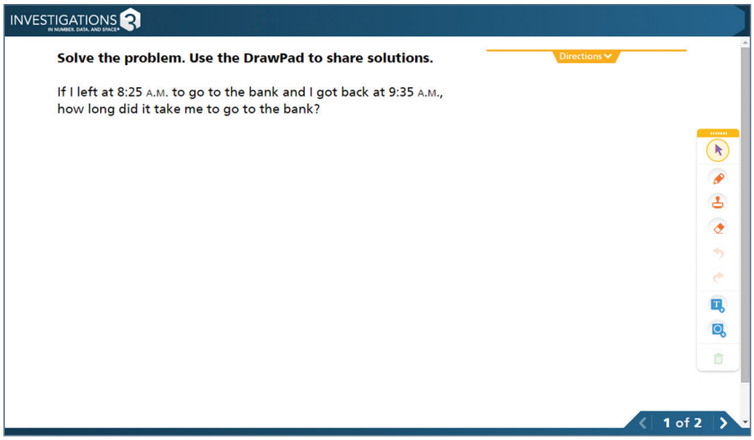
What Time Is It?

10

MATH FOCUS POINTS

- Telling time to the minute on a digital or analog clock
- Determining intervals of time to the minute
- Solving problems involving addition or subtraction of time intervals in minutes

Display the Teacher Presentation or tell students the following story:



If I left at 8:25 A.M. to go to the bank and I got back at 9:35 A.M., how long did it take me to go to the bank?

In pairs, students share their solutions. Ask students to share their solutions with the class, which may include: adding 1 hour to 8:25 and then 10 more minutes to 9:35, or adding 35 minutes to 8:25 and then another 35 minutes. Ask similar questions using 1:42 as the starting time and 2:21 as the ending time.

1 DISCUSSION

What Do You Do with the Extras?

15

MATH FOCUS POINT FOR DISCUSSION

- Making sense of remainders in terms of the problem context

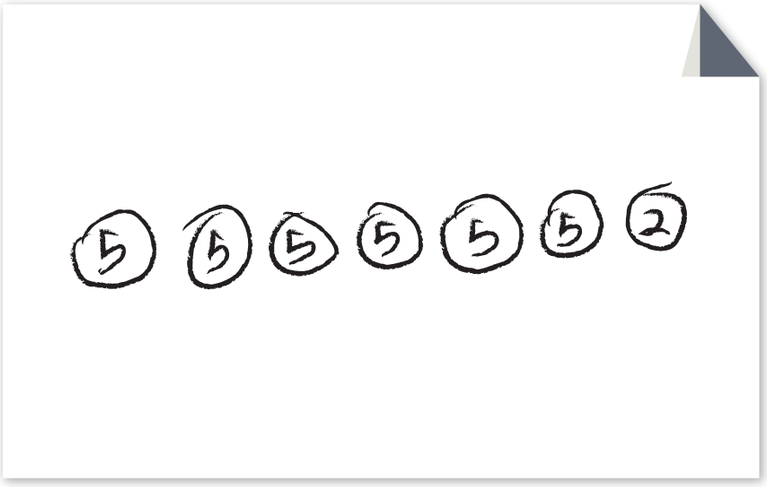
Display Problem 1 on *Student Activity Book* page 475.

What division problem were you being asked to solve for Problem 1? (32 ÷ 5)

Who's willing to explain how they solved this problem?

Ask a volunteer to use a representation to explain how they solved the problem.

Look at this representation. Who can say in words what this representation is showing? (If students seem unsure, ask them how many students are in cars and how many students are left over.)



STUDENTS MIGHT SAY

"There are 6 groups of 5—that shows 6 cars are needed for 30 students. But there are 2 students left."

How many cars are needed for all the students?

STUDENTS MIGHT SAY

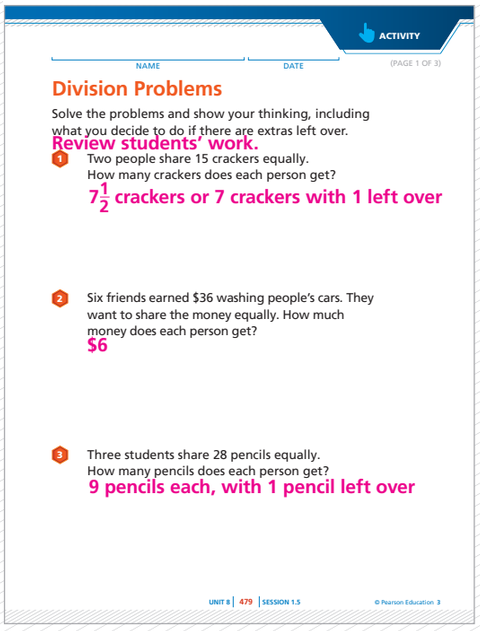
"You need 7 cars! No one would want to be one of the 2 students that are left."

Make sure students agree that seven cars are needed.

Yesterday when you were solving 14 cookies divided among 4 people, you agreed there could be 2 left over or you could divide the 2 extra in half and each person could get $3\frac{1}{2}$ cookies. But if you're dividing 14 balloons among 4 people, you can't cut the balloons in half, so the answer would be 3 balloons per person. What happened in this problem? Was it like the cookies or the balloons, or is it different in some way from those problems?

Give students time to discuss this with their neighbors.

STUDENT ACTIVITY BOOK, P. 479



ACTIVITY

NAME DATE (PAGE 1 OF 3)

Division Problems

Solve the problems and show your thinking, including what you decide to do if there are extras left over.

Review students' work.

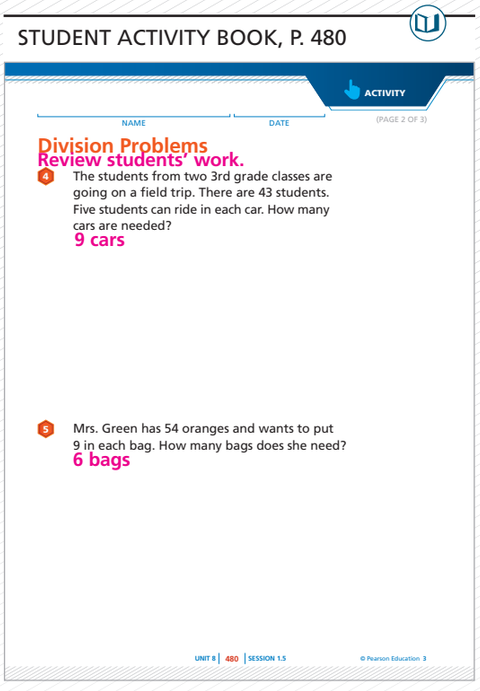
1. Two people share 15 crackers equally. How many crackers does each person get?
   $7\frac{1}{2}$ crackers or 7 crackers with 1 left over

2. Six friends earned $36 washing people's cars. They want to share the money equally. How much money does each person get?
   $6

3. Three students share 28 pencils equally. How many pencils does each person get?
   9 pencils each, with 1 pencil left over

UNIT 8 | 479 | SESSION 1.5 © Pearson Education 3

STUDENT ACTIVITY BOOK, P. 480



ACTIVITY

NAME DATE (PAGE 2 OF 3)

Division Problems

Review students' work.

4. The students from two 3rd grade classes are going on a field trip. There are 43 students. Five students can ride in each car. How many cars are needed?
   9 cars

5. Mrs. Green has 54 oranges and wants to put 9 in each bag. How many bags does she need?
   6 bags

UNIT 8 | 480 | SESSION 1.5 © Pearson Education 3

STUDENTS MIGHT SAY

"Well, it's the same because there are some left over, but then it's different. In those problems you could just ignore what was left, because you were dividing things equally, and it was OK if some were left over."

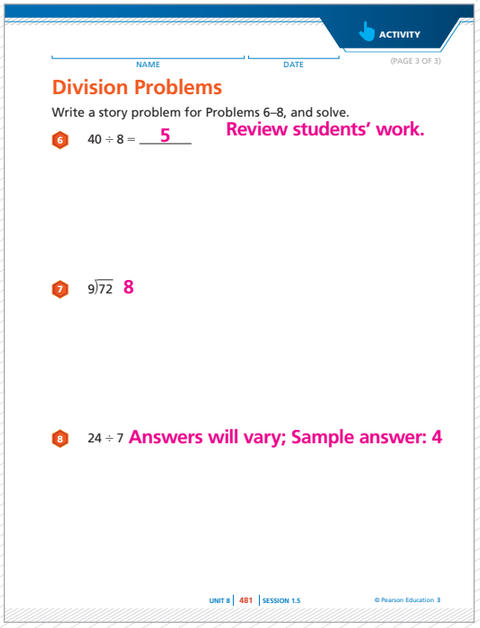
"This time you can't ignore the extra students. They have to go on the field trip too."

"You can't have part of a car. You need one whole extra car even if you don't fill it up."

So it seems really important we think about the story context here, and what makes sense. It's not just about solving the division problem correctly. It's really important that once you have the answer to the division problem, you use those numbers to make sense of what the story context is asking. MPN

Have a similar discussion about Problem 4, using a representation to show the rows and students. Establish that the division problem is 60 ÷ 9, and that the answer to the word problem is 6, because that is how many rows are completely filled.

STUDENT ACTIVITY BOOK, P. 481



ACTIVITY

NAME DATE (PAGE 3 OF 3)

Division Problems

Write a story problem for Problems 6–8, and solve.

6. 40 ÷ 8 = 5 Review students' work.

7. 9)72 8

8. 24 ÷ 7 Answers will vary; Sample answer: 4

UNIT 8 | 481 | SESSION 1.5 © Pearson Education 3

MATH PRACTICE NOTE

MPN **MP6 Attend to precision.** Part of attending to precision is computing carefully, keeping track of all parts of the problem, and double-checking one's work in order to make sure the computation has been carried out accurately. Another part is attending to the requirements of the problem and what kind of precision is needed for the solution. In these problems, students must decide whether the whole-number part of the quotient is an adequate solution, whether the problem situation requires that the quotient be rounded up to the next whole number, or whether fractional parts can be included in the answer.